Inexact inner-outer Golub-Kahan bidiagonalization method: A relaxation strategy

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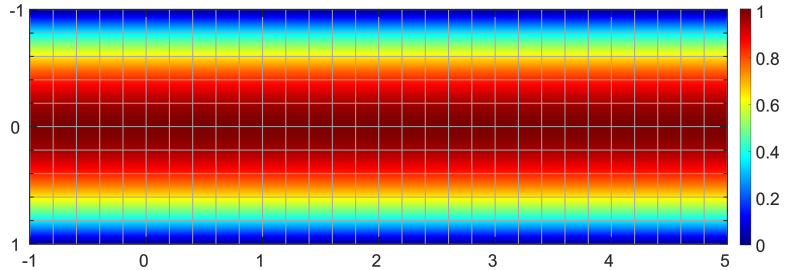
Problem solved: Stokes flow

... in a long and thin rectangular channel:

$$-\Delta\vec{u} + \nabla p = 0$$
$$\nabla \cdot \vec{u} = 0$$



Nested solving and relaxation strategy

Outer solver: GKB

Problem

$$\begin{bmatrix} \mathbf{M} & \mathbf{A} \\ \mathbf{A}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{b} \end{bmatrix}$$

Primal error norm

$$\|e_k\|_{\mathbf{M}}^2 = \sum_{i=k+1}^{n} \zeta_i^2$$

Inner solver: CG

Problem

$$\mathbf{w} = \mathbf{M}^{-1}(\mathbf{A}\mathbf{q}_{k+1} - \beta_{k+1}\mathbf{M}\mathbf{v}_k)$$

Tolerance

$$\tau / f(\zeta)$$

Effective: savings between 30% and 60%.
Simple: no problem dependent parameters are needed.
Negligible cost: only scalar computations are performed.