Sparse-Days 2022

20-22 June 2022

Adaptive Precision Sparse Matrix–Vector Product

and its Application to Krylov Solvers

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Joint work with

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Today's floating-point landscape

		Bits Signif. (t)	Exp.	Range	$u = 2^{-t}$
bfloat16	B	8	8	$10^{\pm 38}$	4×10^{-3}
fp16	H	11	5	$10^{\pm 5}$	5×10^{-4}
fp32	S	24	8	$10^{\pm 38}$	6×10^{-8}
fp64	D	53	11	$10^{\pm 308}$	1×10^{-16}
fp128	Q	113	15	$10^{\pm 4932}$	1×10^{-34}

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- **Great benefits:**
  - Reduced **storage**, data movement, and communications
  - Reduced **energy** consumption (5× with fp16, 9× with bfloat16)
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- **Some limitations too:**
  - Low accuracy (large u)
  - Narrow range

Adaptive precision algorithms

Mix several precisions in the same code with the goal of

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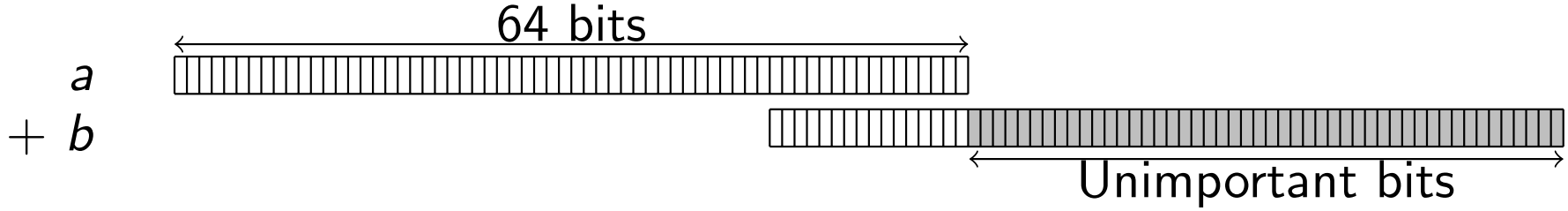
Opportunity for mixed precision:
$\Rightarrow$ **all computations are not equally "important"!**



and small elements produce small errors :

$$|\,\mathrm{fl}(a\,\mathrm{op}\,b) - a\,\mathrm{op}\,b| \leq u|a\,\mathrm{op}\,b|, \qquad \mathrm{op} \in \{+, -, *, \div\}$$

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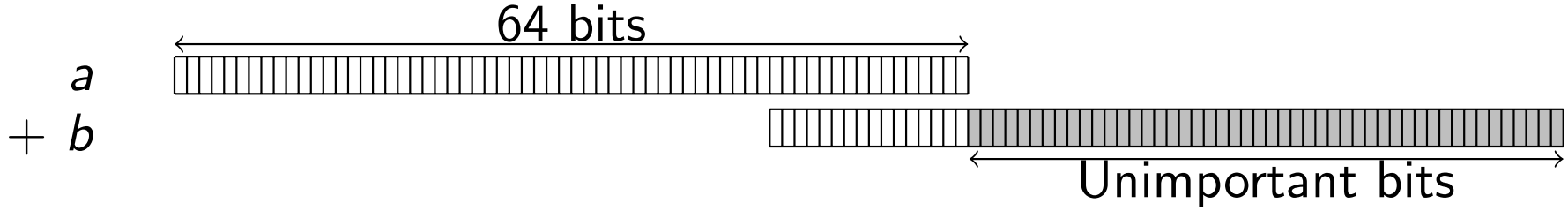

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$\Rightarrow$ We can adapt the precisions to the data at hand

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- **Adaptive precision SpMV**
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  - Deduced algorithm
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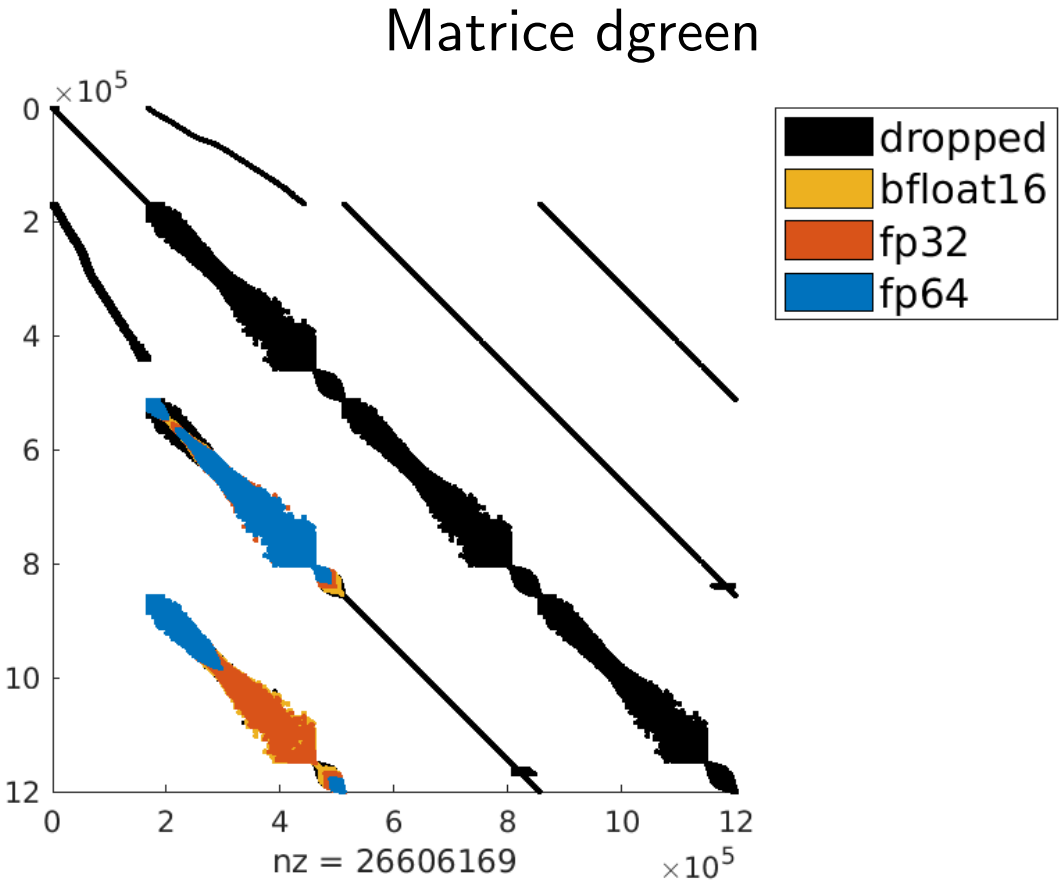
- **Application to Krylov solvers**
  - Importance of SpMV in GMRES and GMRES-IR
  - Convergence scheme

Example of mixed-precision SpMV gains



For some matrices, many elements can be dropped that leads to major gains.

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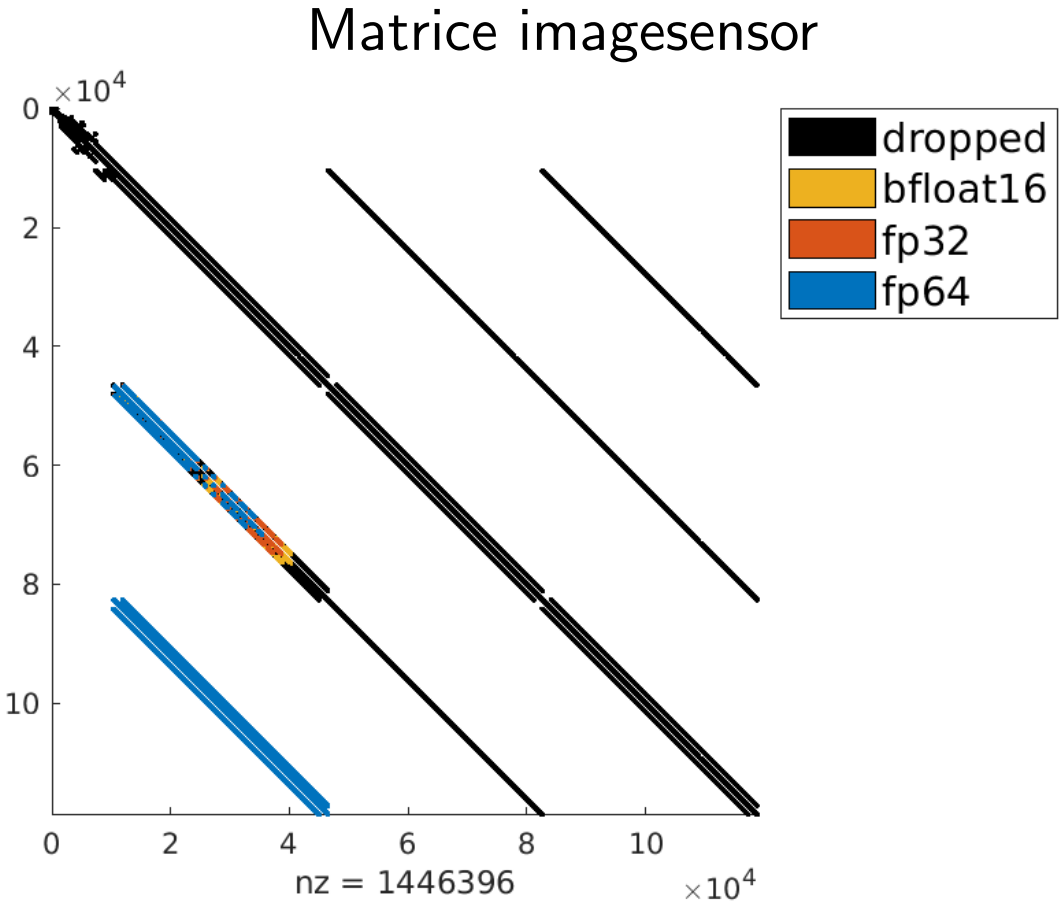

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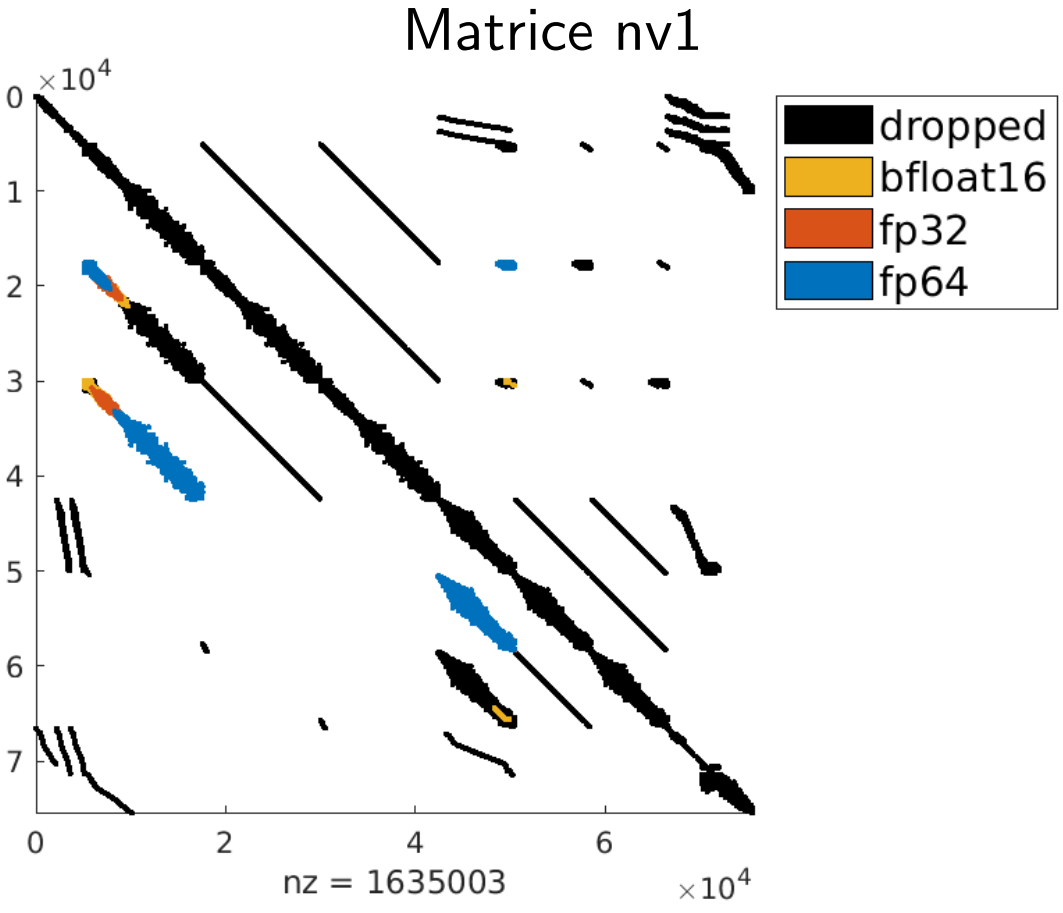

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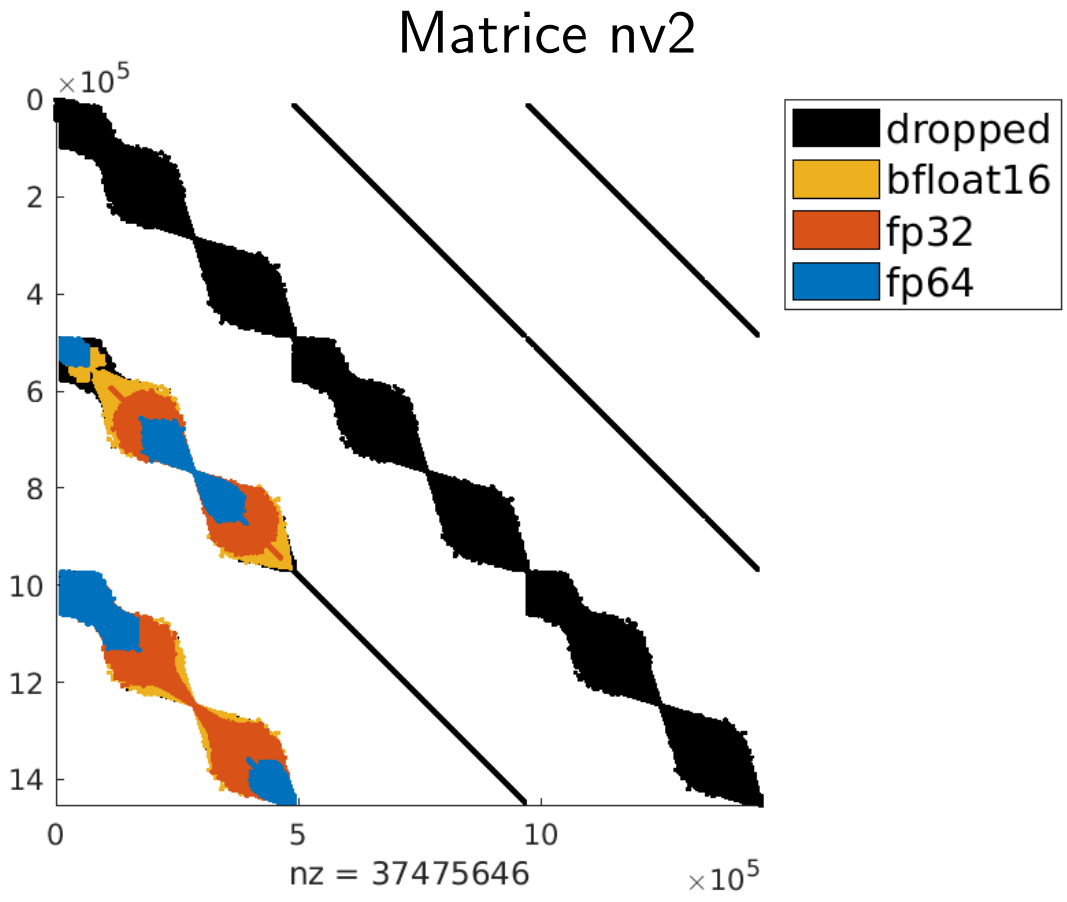

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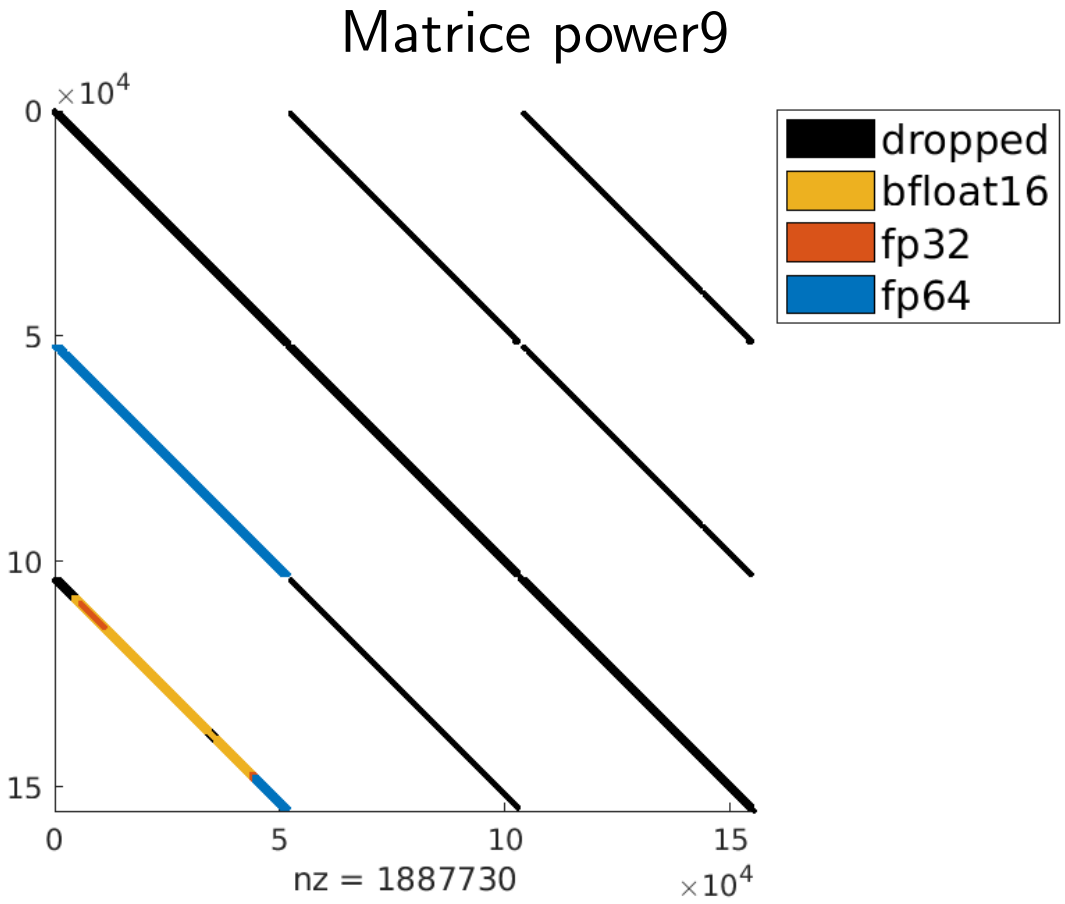

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